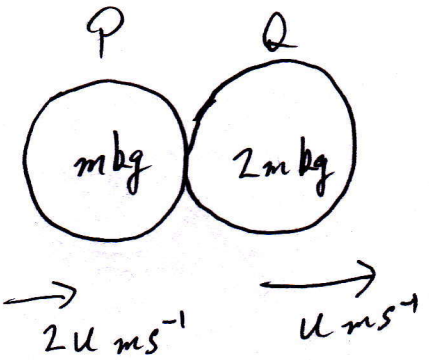
2009 Q5.

(a)



(i) PCM: $m(2u) + 2m(u) = mV_1 + 2mV_2$

$\Rightarrow \quad 4u = V_1 + 2V_2 \quad$ ①

NEL: $V_1 - V_2 = -e(2u - u) \quad$ ②

Substituting: $V_1 - \left(\frac{4u - V_1}{2}\right) = -e(2u - u)$

$\Rightarrow 2V_1 - 4u + V_1 = -2eu$

$\Rightarrow 3V_1 = -2eu + 4u$

$\Rightarrow V_1 = \frac{u(4 - 2e)}{3}$

Similarly $V_2 = \frac{u(4+e)}{3}$

(ii) Speed of Q $= V_2 = \frac{u(4+e)}{3} > u$

since $e > 0$

so Q increases in speed.

(iii) $V_1 = \frac{10u}{9}\ ms^{-1} = \frac{u(4-2e)}{3}$

$\Rightarrow 30u = 9u(4-2e)$

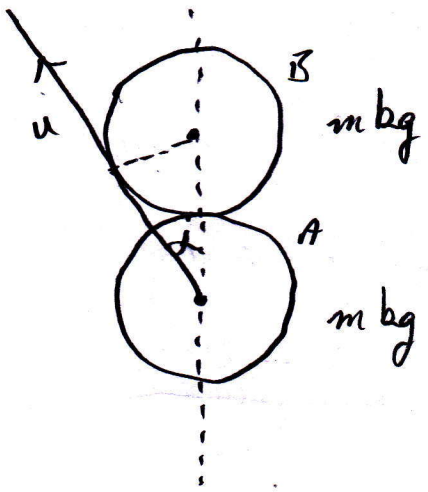
$\Rightarrow \frac{30}{9} = 4 - 2e$

$\Rightarrow \frac{30}{9} - 4 = -2e$

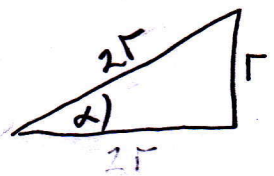
$\Rightarrow -\frac{2}{3} = -2e$

$\Rightarrow \frac{1}{3} = e$

(b) (i)



r = radius of each sphere.



$\sin\alpha = \frac{r}{2r} = \frac{1}{2}$

$\Rightarrow \alpha = \sin^{-1}\frac{1}{2}$

$\Rightarrow \alpha = 30^\circ$

(ii) PCM: $m(u\cos 30) + m(0) = mV_1 + mV_2$

$\Rightarrow \frac{u\sqrt{3}}{2} = V_1 + V_2$ ①

NEL: $V_1 - V_2 = -\frac{4}{5}\left(u\frac{\sqrt{3}}{2} - 0\right)$

$\Rightarrow V_1 - V_2 = -\frac{2\sqrt{3}}{5}u$ ②

$\sin 30^\circ = \frac{1}{2}$
$\cos 30^\circ = \frac{\sqrt{3}}{2}$
$\tan 30^\circ = \frac{1}{\sqrt{3}}$

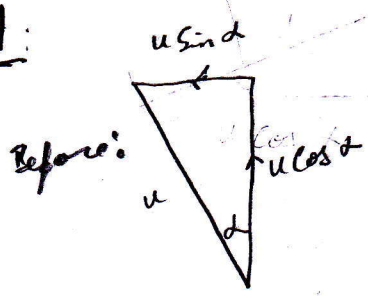
Adding ① and ② $\Rightarrow 2V_1 = \frac{u\sqrt{3}}{2} - \frac{2\sqrt{3}}{5}u$

$\Rightarrow V_1 = \frac{u\sqrt{3}}{4} - \frac{2\sqrt{3}}{10}u$

$\Rightarrow V_1 = \frac{\sqrt{3}}{20}u$

And $V_2 = \frac{9\sqrt{3}}{20}u$

A:



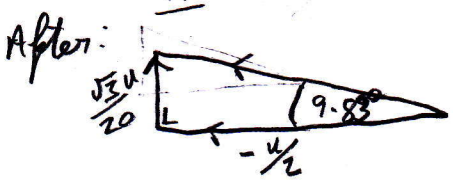
$\Rightarrow$ Velocity A $= u\sin 30°\vec{i} + \frac{\sqrt{3}}{20}u\vec{j}$

$= -\frac{u}{2}\vec{i} + \frac{\sqrt{3}}{20}u\vec{j}$

$\Rightarrow$ Direction A $= \tan^{-1}\left(\frac{\sqrt{3}}{10}\right)$

$= 9.83°$

A:



B: Velocity B $= 0\vec{i} + \frac{9\sqrt{3}}{20}u\vec{j}$

B moves along the lines of their centres.

(iii) KE before $= \frac{1}{2}mu^2 + \frac{1}{2}m(0)^2 = \frac{1}{2}mu^2$

KE After $= \frac{1}{2}m(V_1)^2 + \frac{1}{2}m(V_2)^2$

$= \frac{1}{2}m\left(\sqrt{\left(-\frac{u}{2}\right)^2 + \left(\frac{\sqrt{3}u}{20}\right)^2}\right)^2 + \frac{1}{2}m\left(\frac{9\sqrt{3}}{20}\right)^2$

$= \frac{1}{2}m\left(\frac{u}{4} + \frac{3}{400}u^2 + \frac{243u^2}{400}\right)$

$\Rightarrow$ KE Lost $= \frac{27}{400}mu^2$ $\Rightarrow$ % KE Lost $= \frac{\frac{27}{400}mu^2}{\frac{1}{2}mu^2} \times 100 = 13.5\%$